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One-dimensional stress evaluation of a ring bulk HTS with shrinkage fit by an iron ring

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Abstract

The stress distributions of a ring bulk high-Tc superconductor are studied in the one-dimensional numerical analysis. Boundary condition is derivated under shrinkage fit by an iron ring. Convergences of the solutions are compared with the simple iteration method and the successive approximation method. Maximum hoop stresses are evaluated during the field-cooled magnetization. Differences of the solutions are also discussed between the present and the previous boundary conditions.

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*Keywords:*high-$T_c$superconductor; stress distribuion; shrinkage fit; successive approximation method

1. Introduction

A trapped field magnet is one application of a melt-processed bulk high-Tc superconductor (HTS). A recent application of the trapped field magnet is a micro nuclear magnetic resonance apparatus with a hollow cylindrical bulk HTS [1]. Evaluation of the maximum stress during the magnetization is important to avoid damages of the bulk HTS. The stress distributions in the HTS for open boundary were well evaluated analytically with one-dimensional model by Ren et al. [2]. The author also studied the stress distributions numerically in the axisymmetric three-dimensional analysis with the open and the perfect fixed boundary conditions [3, 4].

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In the paper, one-dimensional stress distributions of a ring bulk HTS are evaluated with consideration of the boundary condition of shrinkage fit by an outer iron ring. Convergences of the solutions are improved by using the successive approximation method. Maximum hoop stresses are obtained as tensile stresses during the field-cooled magnetization. Solutions with the present boundary condition are also compared with those with the previous perfect fixed boundary condition [4].

2. Numerical formulation

In one-dimensional model, following force equation is obtained from force balance for a small element with radial and hoop stresses σ_r, σ_θ ; and radial Lorenz force F_r;

$$\frac{d\sigma_r}{dr}+\frac{\sigma_r-\sigma_\theta}{r}+F_r=0,\quad \sigma_r=\frac{Y}{(1-\nu^2)}\left(\frac{du}{dr}+\nu\frac{u}{r}\right),\quad \sigma_\theta=\frac{Y}{1-\nu^2}\left(\frac{u}{r}+\nu\frac{du}{dr}\right), \tag{1}$$

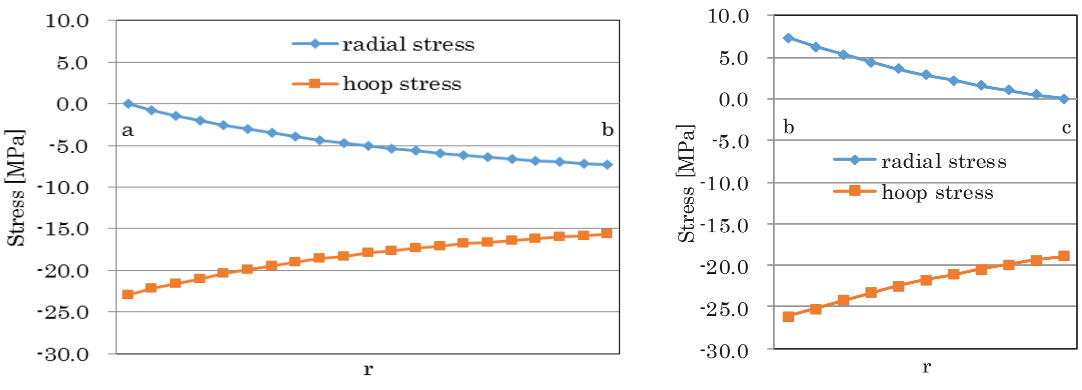
where u is radial displacement and Y and ν are Young's module and Poisson's ratio, respectively. The governing equation is obtained for the radial displacement u, and the radial and the hoop stresses are shown as follows for inner and outer boundary conditions of $\sigma_{r,a}=0$, $\sigma_{r,b}=-p_0$;

$$\frac{d^2u}{dr^2}+\frac{1}{r}\frac{du}{dr}-\frac{u}{r^2}=-\frac{1-\nu^2}{Y}F_r,\quad \sigma_r=-\frac{b^2p_0}{b^2-a^2}\left(1-\frac{a^2}{r^2}\right),\quad \sigma_\theta=-\frac{b^2p_0}{b^2-a^2}\left(1+\frac{a^2}{r^2}\right), \tag{2}$$

where a,b are inner and outer radii. In the one-dimensional combined cylinders model, compressive pressure p_0 by the shrinkage fit of the outer iron ring is shown as follows with shrinkage fit margin δ [5];

$$p_0=\frac{\delta}{b}\frac{Y_1Y_2}{Y_2C+Y_1D},\quad C=\left(\frac{b^2+a^2}{b^2-a^2}+\nu\right), D=\left(\frac{c^2+b^2}{c^2-b^2}-\nu\right), \tag{3}$$

where subscriptions 1,2 of the Young's modules show inner bulk HTS and outer iron ring, respectively. Figs. 1 (a), (b) show the initial stress distributions in the inner bulk HTS and in the outer iron ring. The shrinkage fit margin is set to $\delta=7.5\times10^{-3}$mm so that the compressive pressure p_0 is about 7.35 MPa. The inner and the outer radii of the bulk HTS are a=18.0 mm and b=30.0 mm, and the outer radius of the iron ring is c=40.0 mm. Poisson's ratio ν is treated as 0.3 and Young's modulus of the bulk HTS and the iron ring are set to $Y_1=117\times10^9$ Pa, $Y_2=210\times10^9$ Pa. The stresses in the inner bulk HTS are obtained as compressive stresses in Fig. 1 (a), while the radial stress of the outer iron ring is obtained as tensile stress in Fig. 1 (b).

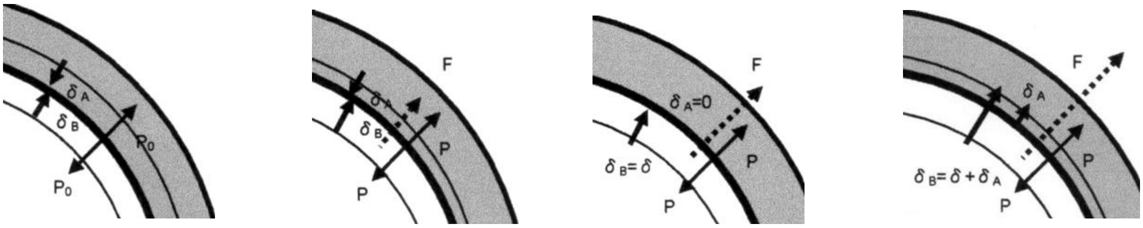


(a) Inner bulk HTS (b) Outer iron ring

Fig. 1. Initial stress distributions.

3. Numerical evaluation with shielding currents

Fig. 2 shows change of shrinkage fit margin δ with increase of shielding current during field-cooled magnetization. In the initial state, the inner bulk HTS shrinks by δ_A and the outer iron ring is extended to δ_B as shown in Fig. 2 (a). The shrinkage fit margin δ becomes $\delta=\delta_A+\delta_B$. When shielding current flows, Lorenz force for the outer radial direction act to extend the outer iron ring. The δ_A becomes small and the δ_B becomes large under the condition of $\delta=\delta_A+\delta_B$ in Fig. 2 (b). After the δ_A becomes zero and the δ_B becomes δ in Fig. 2 (c), both δ_A and δ_B become outer radial direction, and the outer iron ring is extended to $\delta_B=\delta+\delta_A$ in Fig. 2 (d).



(a) zero current, $\delta=\delta_A+\delta_B$ (b) δ_A:small,δ_B:large (c) $\delta_A=0$,$\delta_B=\delta$ (d) $\delta_B=\delta+\delta_A$

Fig. 2. Change of shrinkage fit margin δ with increase of shielding current.

Solutions with shielding currents are obtained by using the finite-difference method and iterative calculations with the boundary condition. Though the solutions are converged more than 100 times of iterations by using the simple iteration method, they are converged about 20 times by using the successive approximation method, where the boundary condition of the n th iterations is approximated as follows ;

$$\sigma_{r,n} = \frac{\sigma_{r,n-1} + \sigma_{r,n-2}}{2}, \quad \text{on} \quad r = b. \tag{4}$$

Fig. 3 shows an example of convergence of the solutions. The solution with the successive approximation method converges fast, while the solution with the simple iteration method vibrates and converges slow.

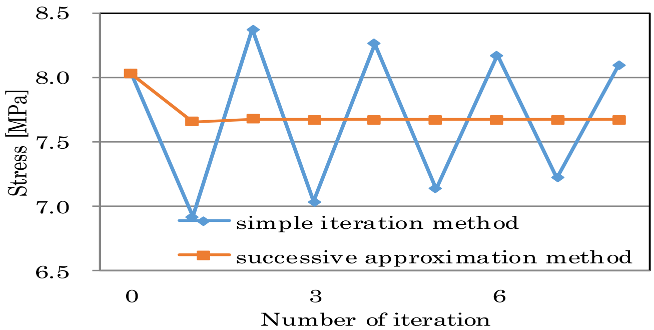
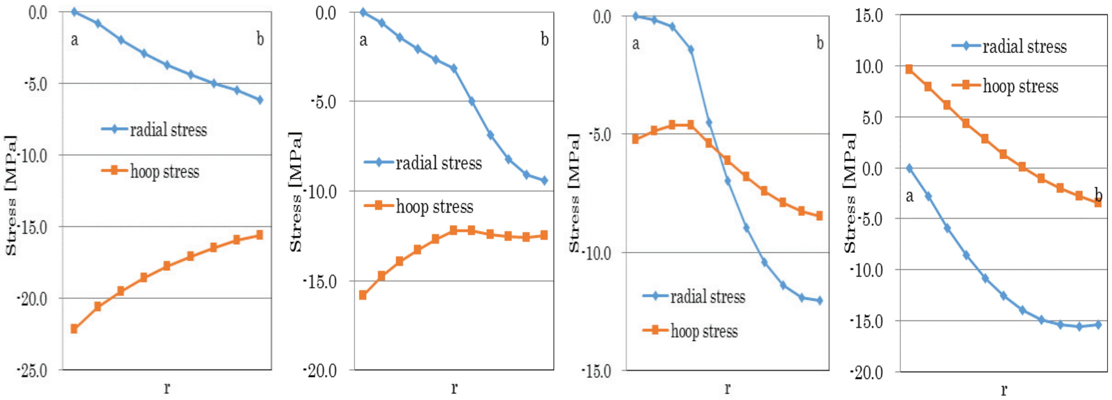


Fig.3. Convergence of solutions in iterations

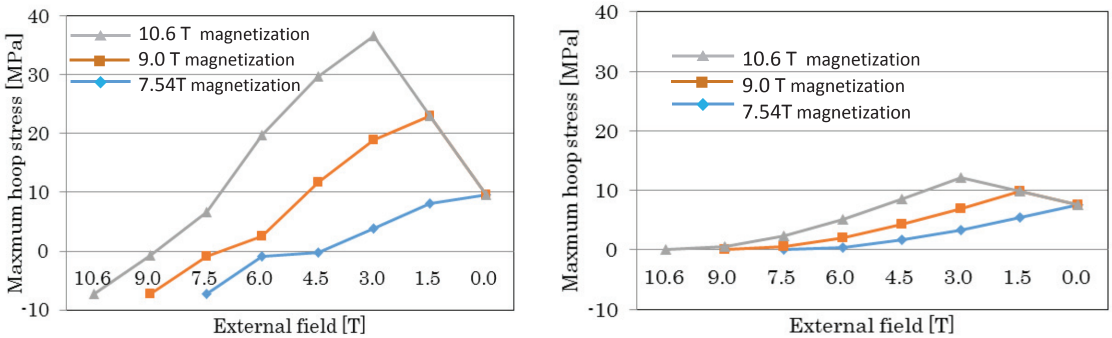
Fig. 4 shows radial and hoop stresses in the ring bulk HTS with shielding currents. The critical current density is set to 5.0×10^8 A/m^2 in the Bean model. The shielding current regions in Fig. 4 (a), (b), (c), (d) are 1/4, 1/2 3/4, full, respectively. The external field is zero so that they correspond to the situations after magnetization. Compressive stresses by shrinkage fit of the outer iron ring are obtained at first as shown in Fig 1. The stress distributions changed with increase of the shielding current distributions in Fig. 4 (a), (b), (c). The hoop stress changes from compressive to tensile with increase of the shielding current region in Fig. 4 (d).

The maximum hoop stresses during the field-cooled magnetization are shown in Fig. 5 (a), (b) for both the present boundary condition and the previous perfect fixed boundary condition. The full magnetization is obtained for $B=\mu_0 J_c(b-a)=7.54$[T] in the present one-dimensional model. There is a peak in the maximum hoop stress during the magnetization, since Lorentz force is induced by the shielding current distribution, the self-field and the external field. The previous perfect fixed boundary condition $u=0$ on $r=b$ is too severe to evaluate the shrinkage fit and small hoop stresses are obtained in Fig. 5 (b).



(a) 1/4 current region (b) 1/2 current region (c) 3/4 current region (d) Full current region

Fig. 4. Radial and hoop stresses in the ring bulk HTS without external field.



(a) Present boundary condition (b) Perfect fixed boundary condition

Fig. 5. Maximum hoop stress during the field-cooled magnetization.

4. Conclusion

The stress distributions of a ring bulk HTS with the boundary condition of shrinkage fit are studied in the one-dimensional numerical analysis. The initial boundary condition with shrinkage fit by an iron ring must be derived from thermal stress analysis to evaluate the practical experimental results. Convergences of the solutions are improved by using the successive approximation method. The axisymmetric three-dimensional analysis will be carried out by using the basic properties of the present one-dimensional solutions.

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